

These questions cover single-step probability, two-step events (with replacement), and mutually exclusive events.

Sample Questions: Probability

Question 1: Probability of Mutually Exclusive Events (Short Answer/Fluency)

A bag contains 10 red marbles, 5 green marbles, and 15 blue marbles. A single marble is drawn randomly from the bag.

What is the probability that the marble drawn is either red or green?

Solution:

1. Calculate the total number of outcomes:

$$\text{Total Marbles} = 10(\text{Red}) + 5(\text{Green}) + 15(\text{Blue}) = 30$$

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2. Calculate the probability of each event:

$$P(\text{Red}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{10}{30} = \frac{1}{3}$$
$$P(\text{Green}) = \frac{5}{30} = \frac{1}{6}$$

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3. Calculate the probability of 'Red or Green' (Mutually Exclusive Events):

$$P(\text{Red or Green}) = P(\text{Red}) + P(\text{Green})$$
$$P(\text{Red or Green}) = \frac{10}{30} + \frac{5}{30} = \frac{15}{30} = \frac{1}{2}$$

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\$\$Final Answer: $\frac{1}{2}$ (or 0.5)

Concept: Calculating the probability of a single event and applying the Addition Rule for mutually exclusive events.

Question 2: Two-Step Probability with Replacement (Multiple Choice)

A standard six-sided die is rolled twice. The outcome of the first roll does not affect the outcome of the second roll.

What is the probability of rolling a number greater than 4 on the first roll AND an odd number on the second roll?

- A. $\frac{1}{6}$
- B. $\frac{1}{3}$
- C. $\frac{2}{3}$
- D. $\frac{5}{6}$

Solution:

1. **Define Event A (First Roll):** Rolling a number greater than 4 ($\{5, 6\}$).

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

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2. **Define Event B (Second Roll):** Rolling an odd number ($\{1, 3, 5\}$).

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

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3. **Calculate the probability of A AND B (Independent Events):**

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B) \\ P(A \text{ and } B) &= \frac{1}{3} \times \frac{1}{2} \\ P(A \text{ and } B) &= \frac{1}{6} \end{aligned}$$

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\$\$Answer: A

Concept: Calculating the probability of two independent events occurring sequentially by multiplying their individual probabilities.

Question 3: Probability Tree Diagram and Conditional Probability (Extended Response)

In a factory, two machines, M1 and M2, produce identical light bulbs. M1 produces 60% of the total output, and M2 produces 40% of the total output.

- 3% of the bulbs produced by M1 are faulty.
- 5% of the bulbs produced by M2 are faulty.

a) What is the probability that a randomly chosen bulb is produced by M1 AND is faulty?

b) What is the overall probability that a randomly chosen bulb from the total output is faulty?

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Solution:

Let's break this down step by step using probability rules and a tree diagram approach.



Probability Tree Structure

We can visualize the process in two stages:

1. Stage 1: Machine Selection

- M1: 60% $\rightarrow P(M1) = 0.6$
- M2: 40% $\rightarrow P(M2) = 0.4$

2. Stage 2: Fault Status

- Faulty given M1: 3% $\rightarrow P(Faulty|M1) = 0.03$
- Faulty given M2: 5% $\rightarrow P(Faulty|M2) = 0.05$

a) Probability that a bulb is from M1 and is faulty

We use the multiplication rule:

$$P(M1 \cap Faulty) = P(M1) \times P(Faulty|M1) = 0.6 \times 0.03 = 0.018$$



Answer: 0.018 or 1.8%

b) Overall probability that a randomly chosen bulb is faulty

We use the **law of total probability**:

$$\begin{aligned} P(Faulty) &= P(M1) \times P(Faulty|M1) + P(M2) \times P(Faulty|M2) \\ &= 0.6 \times 0.03 + 0.4 \times 0.05 = 0.018 + 0.02 = 0.038 \end{aligned}$$



Answer: 0.038 or 3.8%